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# A STUDY OF THE EFFECT OF CROSS BEAMS' DISTANCE ON IMPROVING THE GIRDERS

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A R T I C L E I N F O

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A B S T R A C T

Plate girders are used frequently in major industrial structures. In the systems covered by these types of beams, reduced deflection and decreased maximum bending moment can be considered as a method for improvement. In cross beams or braced beams, element perpendiculars to the girders are provided. In this paper, the effect of cross beams and their stiffness and distance on improvement of girders has been studied. Differential equation of degree four governing the behavior of the partial structures is a quadratic relationship which is determined by solving the mathematical transformations appertaining thereto and the fundamental equations have been provided thereby. The resulting equations have solved a practical problem and the result reached at employing the proposed solution has been modeled in SAP2000. The results show that different values of stress, bending moments and forces in the systems where the covers are improved can be calculated by changing the stiffness and distance of the cross beams.

## 1.0 Introduction

Using plate girders in building construction of about 100m may be economical but, in buildings higher than these spans, it may require use of truss. Web thin flexural members are influenced by the flexural behavior of stiffness and are placed in the cross beam and have the ability to put aside the destroying forces. In two - dimensional structures, the beam for plate industrial floor systems are used. We can add elements and new components perpendicular to the bearing element in a two-dimensional structure so the entire stiffness of structure will be increased. These members can be located either horizontally or vertically between the existing structures and transform their behavior to a spatial or three dimensional one. By creating three-dimensional structures, the upper connections in two directions perpendicular to communicate and connect parts will be available.

Since 1960, extensive researches were started on the slender girders in USA and Japan and have lasted in the past several decades. Sadovsky et al. (2005) in their researches focused on a method to determine the base principle of stable energy to measure the degree of distortion and deformation effects of errors. However, some aspects of the issue is still unresolved. Hu and Cui (2003) compared the ultimate strength of reinforced and non-reinforced structures by simple analytical formulas designed to measure final strength of beams. Paik et al. (2003) designed a method for the treatment of reinforced plastic plate under different loads until reaching their final strength.

Changes in the behavior of a two-dimensional structure to a space structure, leads to increased stiffness in the whole space and accordingly, deflection of structure is reduced. This subject is one of the key points in examining the linear elastic behavior of the steel structure in relation to the elements in the project resistant structures which will be paired and investigated with a particular perspective in this paper.

## 2.0 The Governing Differential Equation

Transverse (inhibitory) beam which are perpendicular to each other and are located in a determined distance equal to the original envelope girder or trusses are shown in figure 1. We assume profile stiffness and load of them are equal. Reaction forces exerted by the main beams onto the transverse beams are as Figure 2 which shows a transverse optional beam at the crossing point, so the deformation differential equation can be written as (Chen and Lui, 1987):

$$Z_i = +\beta \frac{QL^3}{EI_i} - \alpha \frac{R_j L^3}{EI_j} \quad (1)$$

$Z_i$: deflection of the girder or truss, Q: loads of the main beam, $R_j$: reaction of transverse beam in response of the main beam, E: modulus elasticity of materials. $I_i$: Moment of inertia for main beam, $\alpha$ and $\beta$ are coefficients related to the profile of times. such coefficients can be pre-set according to the type of load and rest points.

If the primary beam number will not be less than 5, it will still be possible to substitute single recipe with massive loads (Araeb, 1990) ie $q=R_j/a$ q: a massive load of intensity

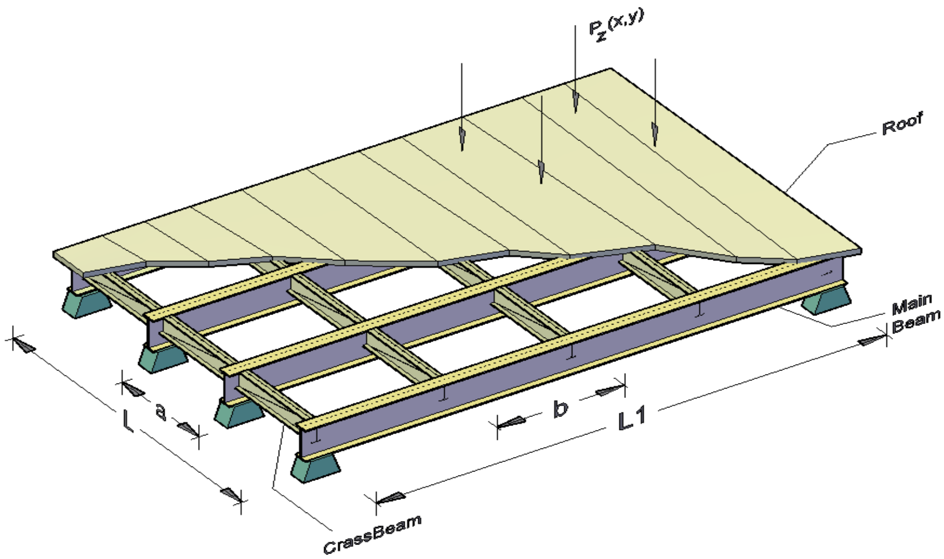



**Figure 1(a)**: Girders in roof systems, cross beam reactions on the main beam (main girders)

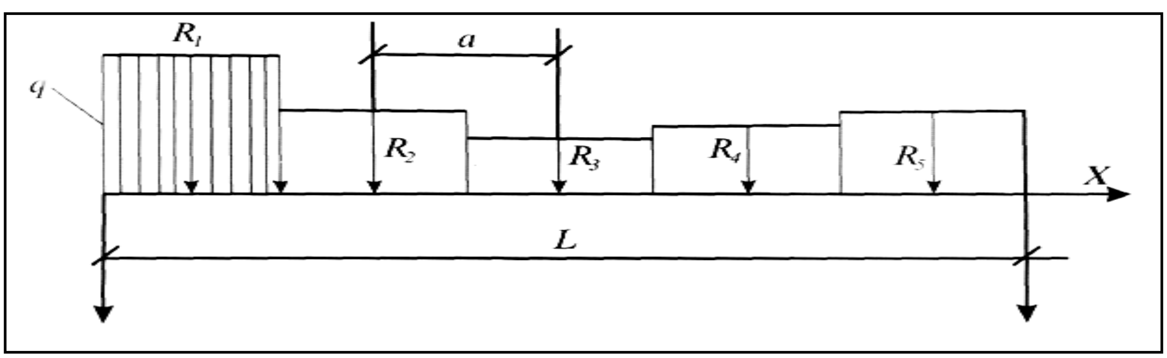



**Figure 1(b)**: Girders in roof systems, cross beam reactions on the main beam (secondary girders)

Thus, the existing beams (main beams) are considered in the order of *i* shot at the intersection of the main beam transverse nodes *j* and, optionally, a set of a.b are obtained from Equation (2):

$$Z_i = +\beta_i \frac{QL^3}{EI_i} - \frac{L^3}{EI_j} \sum_{j=1}^{m} \alpha_j . R_j \quad (2)$$

In the above equation, m is the number of beams or trusses to add to the cross section of roof system. By placement and simplifying, we will have:

$$q = \frac{d^2M}{dx^2} = EI_j\frac{d^4z}{dx^4} \quad , \quad R_j = aq = aE\dot{I}_j$$

$$Z_i = +\beta_i\frac{QL^3}{EI_i} - \frac{a\,L^3}{EI_j}\sum_{j=1}^{m}\alpha_j EI_j\left(\frac{d^4z}{dx^4}\right) \tag{3}$$

**3.0 Mathematical Conversions**

Understanding Equation (3) is rather difficult and complicated and mathematical transformations can be used to solve it. If we assume that in both sides the numbers of beams (or trusses) are high and their stiffness in both directions are equal, then solving the problem would be more easy and solving differential equation of degree 4 will continue as follow:
According to Figure 1 for each node in the intersection of the beams, the amount of load will be as $q.a.$b . In general cases of q(x, y), node load at the intersection of the incoming beam is split into two parts. The R-value for the main beams is parallel to the x-axis and y-axis is parallel to the transverse beam of R and we will have:

$$R + \bar{R} = q(x, y)ab \tag{4}$$

$$R = aEI_x\frac{\partial^4\omega}{\partial x^4} \;;\quad \bar{R} = bEI_y\frac{\partial^4\omega}{\partial y^4}$$

By substituting R and $\bar{R}$ into Equation (4) we will have:

$$EI_x\frac{\partial^4\omega}{\partial x^4} + bEI_y\frac{\partial^4\omega}{\partial y^4} = q(x, y)ab \tag{5}$$

Considering the size of the system created in two directions x and y, and simulating a structural steady, consistent performance space in three-dimensional format, we can skip the grid ceiling system and an integrated suite can be written as (Makowski, 1987; Alinia and Kashizadeh, 2006):

$$\omega = f\sin\frac{\pi x}{l}.\sin\frac{\pi y}{l_1} \tag{6}$$

ω: deflection in any part of the roof, f: deflection in the middle of the roof, $l_1$ and l: length of main and cross beams.
Ordinary differential equations can be obtained by replacing relation (5) in Equation (6) whose results will be equal to the solution of the differential Equation (3). But before continue solving, the following mathematical transformations are required to apply. In general cases, Equation (7) can be written as:

$$\omega = \sum X_n(x)Y_n(y) \tag{7}$$

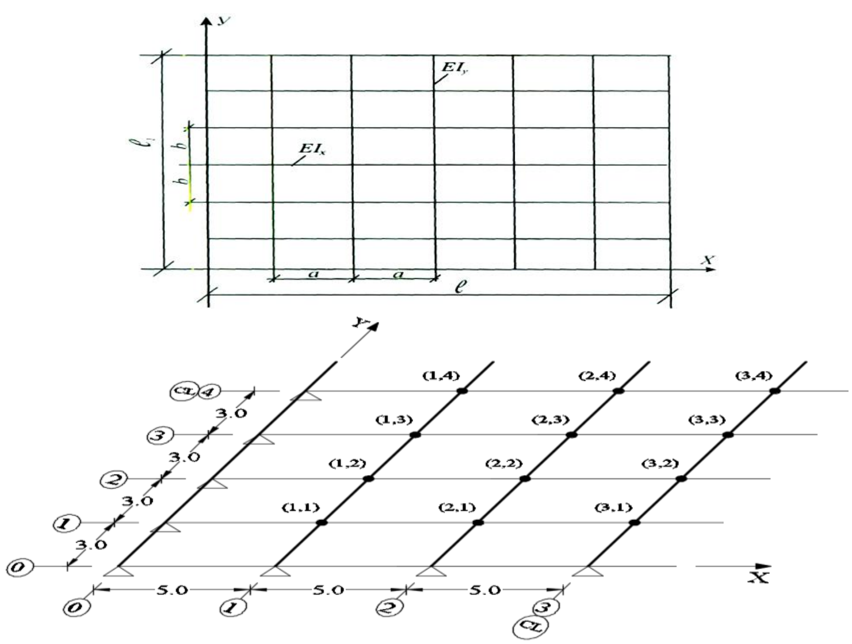



**Figure 2**: Main grid ceiling system with the secondary added beams

Derivate of function $X_n$ (x) will change with respect to the constant factor, in other words, their properties change iteratively and can be written as follow:

$$\frac{d^4X_n(x)}{dx^4} = \eta_n^4 X_n(x) \tag{8}$$

Using orthogonal properties of these relationships, it is possible to convert the right side of Equation (5) to form a series of related mathematical functions of $X_n$ (x) and then, write it as follow:

$$q(x,y)ab = b\sum P_n(y)\,X_n(x) \qquad (9)$$

$P_n$ (y) is the arguments function of y. To determine the expression $P_n$ (y) , we can multiply $X_n$ (x) to both sides in Equation (9) and integrate it as follow:

$$\int_0^l q(x,y)abX_n(x)dx = b\sum P_n(y)\int_0^l X_n(x)X_n(x)dx$$

Here for $X_n$ (x) orthogonal properties are considered and can write:

$$P_n(y) = a\frac{\int_0^l q(x,y)X_n(x)dx}{\int_0^l X_n^2(x)dx} \qquad (10)$$

By replacement of relation (5) in Equation (4) and taking into account the relationship (7), the below equation will be achieved:

$$\sum X_n(x)\left[bEI_yY_n^{IV}(y) + \eta_n^4 aEI_xY_n(y) - P_n(y)b\right] = 0 \quad (11)$$

$$E\dot{I}_yY_n^{IV}(y) = P_n(y) - k_nY_n(y)$$

$$k_n = EI_x\eta_n^4\frac{a}{b} \qquad (12)$$

Equation (11) is the relation of reaction beam on elastic foundation in which parameter $Y_n$ can be determined from the condition of an anchor. Thus, instead of applying complicated Equation (3) we can use ordinary differential Equations (8) and (11) to determine the values of $X_n$ (x) and $Y_n$ (y) by substituting them in Equation (7), the values of ω will be calculated. The obtained values will differentiate two or three times with respect to x and y and the bending moment and shear force are calculated in both directions, then the stress levels in a two-dimensional roof, can be obtained and by choosing transverse beams with appropriate distances and adjusting stiffness, applied stress to the main beam will be obtained .

## 4.0 Application of Relations

Characteristic relation of Equation (8) will be as follow:

$$\lambda^4 - \eta_n^4 = 0$$

The roots of the equation are $\lambda_1 = \eta_n$ ; $\lambda_2 = -\eta_n$ ; $\lambda_3 = i\eta_n$ ; $\lambda_4 = -i\eta_n$ . Private solutions are as follow: $e^{\eta_n x}$ ; $e^{-\eta_n}$ ; $e^{i\eta_n^x}$; $e^{-i\eta_n^x}$ .

The combination of private answer with general solutions in the so called equation will be as follow:

$$X_n(x) = A\,ch\,\eta_n^x + B\,sh\,\eta_n^x + C\cos\eta_n^x + D\sin\eta_n^x \quad (13)$$

By integrating from Equation (14) and applying the boundary conditions, we can determine the coefficients of Equation (15) and then by applying mathematical transformations and integration, we have:

$$\frac{d^4Y_n(y)}{dy^4} + 4\beta_n{}^4Y_n(y) = \frac{P_n(y)}{EI_y} \qquad (14)$$

$$\beta_n{}^4 = \frac{k_n}{4EI_y}\ ;\ \beta_n = \sqrt[4]{\frac{a}{48}\frac{EI_x}{EI_y}}\cdot\frac{n\pi}{l} \qquad (15)$$

To solve Equation (17) we used hyperbolic functions and optional requirements for simplified integration are as follow:

$$\bar{A}_{0n} = -\frac{aq}{n\pi\beta_n{}^4EI_y}$$

(16)

$$\bar{B}_{0n} = -\frac{aq}{n\pi\beta_n{}^4EI_y}\cdot\frac{\sin 2\beta_nl_1 - 2chs\beta_nl_1}{\cos 2\beta_nl_1 - ch2\beta_nl_1}$$

$$\bar{C}_{0n} = -\frac{aq}{n\pi\beta_n{}^4EI_y}\cdot\frac{sh\,2\beta_nl_1 - 2shc\beta_nl_1}{\cos 2\beta_nl_1 - ch2\beta_nl_1}$$

Finally, the general solution of the deflection equation will be as follow:

$$\omega = \sum_{n=1}\sin\frac{n\pi}{l}x\left(\frac{aq}{n\pi^4\beta_n{}^4EI_y} + \bar{A}_{0n}chc\beta_ny + \bar{B}_{0n}chs\beta_ny + \bar{C}_{0n}shc\beta_ny\right) \qquad (17)$$

Here

$$chc\beta_ny = ch\beta_ny\cdot\cos\beta_ny$$
$$chs\beta_ny = ch\beta_ny\cdot\sin\beta_ny$$
$$shc\beta_ny = sh\beta_ny\cdot\cos\beta_ny$$
$$shs\beta_ny = sh\beta_ny\cdot\sin\beta_ny$$

Using the *relations* $M_x$=-$EI_x$ ω''$_x$, $Q_x$=-$EI_x$ ω'''$_x$ *and* $M_y$ =-$EI_y$ ω''$_y$, $Q_y$=-$EIy$ ω'''$_y$ it can be shear the applied moments to determine the values of the reaction of rely and also calculate the confluence of the main and cross beams.

## 5.0 Problem Solving

To plan and cross section shown in Figure 2, and cross section girder in Figure 2 to regulate stresses, beams with a length of 30 m in the x beam transverse (strengthening) and of length L = 24 m in the y are considered in this special situation. Beam distances are considered 3 meters in length direction and 5 meters in the cross direction .Flexural stiffness is assumed to be equal in two directions. The load of the roof covering is equal to q = 5 kN\m$^2$. The state of reducing deflection and bending moment of roof are compared

in two states: one in existing transverse beams and another, without them.
For various nodes, according to the relations given in this paper, the amount of nodes are calculated using the Excel computer program as well as the modeling (SAP2000).

**Table 1:** Calculating the Deflection and Moment at the Junction of the Main and Secondary Beams

$$\beta_n = \sqrt[4]{\frac{a}{48}\frac{EI_x}{EI_y}} \cdot \frac{n\pi}{l}$$

***Equation (18)***

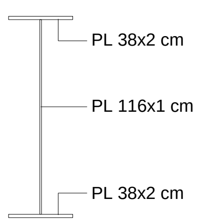
$\beta_1 = 0,0842$ $\beta_3 = 0,2526$ $\beta_5 = 0.421$ ${\beta_1}^2 = 0.00709$ ${\beta_3}^2 = 0.0638$ ${\beta_5}^2 = 0.1772$

${\beta_1}^4 = 5 \times 10^{-5}$ ${\beta_3}^4 = 4.07 \times 10^{-3}$ ${\beta_5}^4 = 3.1414 \times 10^{-2}$

***Equation (19)***

| | | |
|---|---|---|
| $\bar{A}_{01} = -\frac{159155}{EI_y}$ | $\bar{A}_{03} = -\frac{652}{EI_y}$ | $\bar{A}_{05} = -\frac{51}{EI_y}$ |
| $\bar{B}_{01} = -\frac{42097}{EI_y}$ | $\bar{B}_{03} = -\frac{70}{EI_y}$ | $\bar{C}_{05} = \frac{51}{EI_y}$ |
| $\bar{C}_{01} = \frac{173298}{EI_y}$ | $\bar{C}_{03} = \frac{649}{EI_y}$ | $\bar{C}_{05} = \frac{51}{EI_y}$ |

***Sections Properties*:**




$I_x$=653100 $cm^4$
E=2,1×$10^4$ kN /$cm^2$

$\omega_{1,1} = \omega_{1,5} = 1.25$ cm

***Equation (20)***

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1.25 | 2.10 | 2.4 | 2.1 | 1.25 |
| 2 | 2.20 | 3.85 | 4.41 | 3.85 | 2.2 |
| 3 | 2.94 | 4.99 | 5.11 | 4.99 | 2.96 |
| 4 | 3.13 | 5.32 | 5.85 | 5.32 | 3.13 |
| 5 | 2.94 | 4.99 | 5.11 | 4.99 | 2.2 |
| 6 | 2.2 | 3.85 | 4.41 | 3.85 | 2.2 |
| 7 | 1.25 | 2.10 | 2.4 | 2.10 | 1.25 |

$M_{x(1,1)} = M_{x(1,5)} = 224$ kN.m

***Equation (21)***

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 224 | 292 | 314 | 292 | 224 |
| 2 | 391 | 543 | 282 | 543 | 391 |
| 3 | 489 | 707.5 | 766 | 707.5 | 482 |
| 4 | 579 | 754 | 824 | 754 | 579 |
| 5 | 486 | 707.5 | 766 | 707.5 | 482 |
| 6 | 391 | 543 | 585 | 543 | 391 |
| 7 | 224 | 292 | 314 | 292 | 284 |

Table 1 shows the maximum deflection of ω and maximum moment of $M_x$ at nodes (4,3) in the middle of the roof syste. *in simple case*

$$\omega = \frac{5ql^4}{384EI_y} = 11.42 \text{ cm}$$

and

$$M_{max} = \frac{ql^2}{8} = 1675.8 \quad \text{KN.m}$$

The results computed by SAP 2000 for deflections are shown in Figure 3 (U3=5.81 cm).

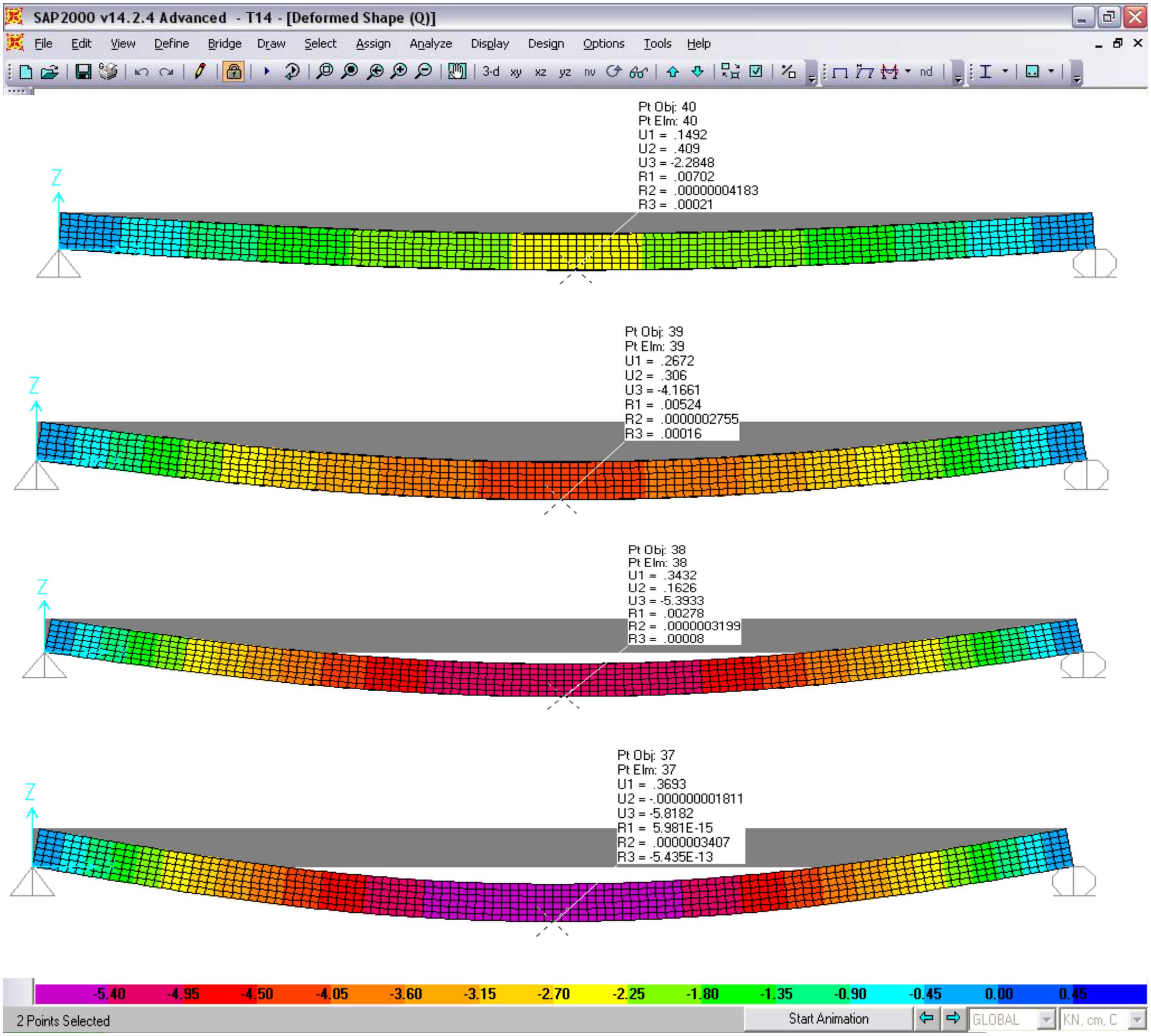



**Figure 3**: Computed results for deflections by SAP2000

In Table 2 the results of analysis achieved by computer program SAP2000 are compared with the results of the method presented in this paper. The results show an excellent compatibility of two methods which shows the maximum deflection of ω and maximum moment of $M_x$ at nodes (4,3) or in the middle of the roof system the digits have declined 50% in strengthen state in comparison to non-strengthen one.

**Table 2:** Comparing the proposed method with the results from SAP2000

| Node | ω | | | $M_x$ | | | $M_y$ | | | Calculation method` |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | |
| 1 | 1.25 | 2.10 | 2.40 | 224 | 292 | 314 | 345 | 545 | 612 | Analytical |
| | 1.19 | 2.00 | 2.28 | 218 | 290 | 308 | 336 | 537 | 610 | SAP 2000 |
| 2 | 2.20 | 3.85 | 4.41 | 391 | 543 | 585 | 537 | 903 | 1024 | Analytical |
| | 2.16 | 3.65 | 4.17 | 390 | 537 | 578 | 531 | 897 | 1018 | SAP 2000 |
| 3 | 2.94 | 4.99 | 5.11 | 489 | 708 | 766 | 634 | 1045 | 1265 | Analytical |
| | 2.78 | 4.71 | 5.39 | 483 | 701 | 762 | 625 | 1038 | 1259 | SAP 2000 |
| 4 | 3.13 | 5.32 | 5.85 | 579 | 754 | 824 | 666 | 1160 | 1335 | Analytical |
| | 3.00 | 5.08 | 5.82 | 574 | 752 | 821 | 661 | 1156 | 1331 | SAP 2000 |

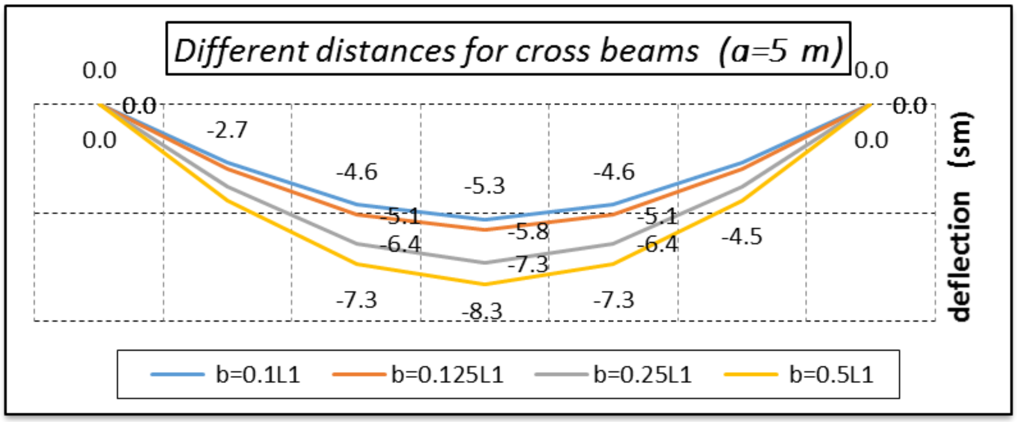



**Figure 4**: The effect of stiffness transverse element

In Figure 4, the amount of deflection in accordance with other constant parameters is calculated; it is shown that by reducing the distance between cross beams, their effectiveness in reducing deflection becomes higher.

### 6.0 Conclusion

Increasing the number of cross beams and reducing the distance between them decreases their impact on economic development. The optimal distance for the cross beam can be determined for each type of roof system.

In the proposed method, it can be calculate the amount of reduced stress and bending moments by changing several affecting parameters and achieve an adjustment of optional stress. Other factors, including the backing, boundary conditions, a subsidiary of loading and shooting the junction depth of the main beam are effective in regulating stress. Other results that can be referred to are as follow:

i. By means of determining the differential equation governing the behavior of the main and cross beams at the intersection, adding to the roof load bearing system and converting it to an ordinary differential equations and then providing analytical solution, accurate results are obtained whose results were compared with the results of computer detailed analysis, the final results of both methods are equal precisely.

ii. It can be reduce the amount of applied work by maintaining the load pressure constant and executing the appropriate technology.

iii. It can be increase the input load on the roof structure by converting the behavior of structure from two-dimensional to three-dimensional one and keeping the bending moment and shear force as constant amounts.

iv. In roof coverings, the physical and geometrical characteristics of beams and secondary reinforcement have a direct effect on adjusting tensions.

**References**


Sadovský Z., Teixeira A.P., Soares, C.G. (2005), Degradation of the compressive strength of rectangular plates due to initial deflection, *Thin-Walled Structures*, 43(1), 65-82

Paik J.K., Lee J.M., Lee D.H. (2003), Ultimate strength of dented steel plates under axial compressive loads, *International Journal of Mechanical Sciences*, 45(3), 433–448

Hu Y., Cui W.C. (2003), A simplified analytical method to predict the ultimate strength of stiffness plates under combined loading including edge shear, *Journal of Ship Mechanics*, 7(6),60-74

Chen W.F. Lui E.M. (1987) “Structural Stability (Theory and Implementation)”, Elsevier Science

Агаев Н.Г.(1990)- Инженерные методы решения задач устойчивости тонкостенных конструкций(монография).М. Стройиздат, 192 с

Makowski, Z.S. (1981) *Analysis, design and construction of double layer grids*, Applied Science Publishers, pp.1-55.

Alinia M.M. and Kashizadeh S. (2006), Effects of double layer space truss domes, *Journal of Construction Steel Research*, 375-382

SAP2000 Nonlinear Version 10. 0. 1, structural Analysis program, Berkeley (CA), Computers and Structures Inc.